28 Calendar

Introduction

"Today is 15 August 1995". And you are asked to find the day of the week on 15 August 2001.

If you don't know the method, it will prove a tough job for you. This type of question is sometimes asked in competitive exams. The process of finding it lies in obtaining the number of **odd days**. So, we should be familiar with **odd days**.

The number of days more than the complete number of weeks in a given period, are called **odd days**. For example:

(1) In an oridinary year (of 365 days) there are 52 weeks and **one odd day**.

(2) In a **leap year** (of 366 days) there are 52 weeks and **two odd days**.

What is Leap and Ordinary Year?

Every year which is exactly divisible by 4 such as 1988, 1992, 1996 etc. is called a **leap year.**

Also every 4th century is a **leap year**. The other centuries, although divisible by 4, are not leap years. Thus, for a century to be a leap year, it should be exactly divisible by 400. For example:

(1) 400, 800, 1200, etc are leap years since they are exactly divisible by 400.

(2) 700, 600, 500 etc are not leap years since they are not exactly divisible by 400.

How to find number of odd days:

An ordinary year has 365 days. If we divide 365 by 7, we get, 52 as quotient and 1 as remainder. Thus, we may say that an ordinary year of 365 days has 52 weeks and 1 day. Since, the remainder day is left odd-out we call it **odd day**.

Therefore, an ordinary year has 1 odd day.

A leap year has 366 days, i.e. 52 weeks and 2 days.

Therefore, a leap year has 2 odd days.

A century, ie, 100 years has:

76 ordinary years and 24 leap years.

$= [(76 \times 52) \text{ weeks} + 76 \text{ days}] +$
$[(24 \times 52) \text{ weeks} + 24 \times 2 \text{ days}]$

$= 5200 \text{ weeks} + 124 \text{ days}$

$= 5200 \text{ weeks} + 17 \text{ weeks} + 5 \text{ days}$

$= 5217 \text{ weeks} + 5 \text{ days}$

Therefore, 100 years contain 5 odd days.

Now, (i) 200 years contain $5 \times 2 = 10$, ie, 3 odd days.

(ii) 300 years contain $5 \times 3 = 15$, ie, 1 odd day.

(iii) 400 years contain $5 \times 4 + 1 = 21$, ie, no odd day.

Similarly, 800, 1200 etc contain no odd day.

Note: (i) $5 \times 2 = 10$ days = 1 week + 3 days ie. 3 odd days.

(ii) $5 \times 3 = 15$ days = 2 weeks + 1 day ie. 1 odd day.

(iii) 400th year is a leap year therefore, one additional day is added.

(v) Remember all the lines which are given in italics.

Rule 1

To find the day of a week by the help of the number of odd days, when reference day is given:

Suppose a question like "Jan 1, 1992 was a Wednesday. What day of the week will it be on Jan 1, 1993"?

If you recall, 1992 being a leap year, it has 2 odd days. So the required day will be two days beyond Wednesday, that is, it will be 'Friday'.

Working Rule

(i) Find the net number of odd days for the period between the reference date and the given date. Exclude the reference day but count the given date for counting the number of net odd days.

(ii) The day of the week on the particular date is equal number of net odd days ahead of the reference day (if the reference day was before this date) but behind the reference day (if this date was behind the reference day).

Illustrative Examples

Ex. 1: Jan 5, 1991 was a Saturday. What day of the week was on March 3, 1992?

Soln: 1991 is an ordinary year, hence it has only 1 odd day. Thus, Jan 5, 1992 was a day beyond Saturday, ie, Sunday.

Now, in Jan 1992 there are 26 days left, ie, 5 odd days. In Feb 1992 there are 29 days, ie 1 odd day. In March 1992 there are 31 days, ie, 3 odd days.

∴ Total no. of odd days after Jan 5, 1992
= 5 + 1 + 3 = 9 days, ie, 2 odd days.
Therefore, 3 March 1992 will be 2 days beyond Sunday, ie, Tuesday.

Other Method

Total no. of days between Jan 5, 1991 and March 3, 1992 = 360 days in 1991 + (31 + 29 + 3) days in 1992 = 3 odd days.
Therefore, March 3, 1992 is three days beyond Saturday, ie, Tuesday.

Ex. 2: Monday falls on 4th April, 1988. What was the day on 3rd Nov 1987?

Soln: No. of days between 3rd Nov. 1987 and 4th April 1988 = 27 (Nov) + 31 (Dec) + 31 (Jan) + 29 (Feb) + 31 (Mar) + 4 (Apr) = 153 days = 21 weeks + 6 days = 6 odd days
Then, 3rd Nov 1987 was 7 – 6 = 1 day beyond the day on 4th April, 1988. So, the day was Tuesday.

Note: 3rd Nov 1987 lies before 4th Apr 1988, hence the required day will be 6 days before Monday or 7 – 6 = 1 day beyond Monday

Ex. 3: Today is 21st August. The day of the week is Monday. This is a leap year. What will be the day of the week on this day after 3 years?

Soln: Since this is a leap year, none of the next 3 years is a leap year. So, the day of the week will be 3 days beyond Monday, ie, it will be Thursday.

Ex. 4: It was Thursday on 2nd Jan 1993. What day of the week will be on 15th March 1993?

Soln: Total no. of days = 29 (Jan) + 28 (Feb) + 15 (Mar)
= 72 days = 10 weeks + 2 days = 2 odd days.
Thus, the given date will fall on two days beyond Thursday, ie, Saturday.

Exercise

1. On what date (among the following) of August 1980 did Monday fall?
   a) 18th b) 16th c) 24th d) 12th
2. January 1, 1992 was a Wednesday. What day of the week will it be on January 1, 1993?
   a) Monday b) Tuesday c) Sunday d) Friday
3. On January 12, 1980, it was Saturday. The day of the week on January 12, 1979 was:
   a) Saturday b) Friday c) Sunday d) Thursday
4. On July 2, 1985, it was Wednesday. The day of the week on July 2, 1984 was:
   a) Wednesday b) Tuesday c) Monday d) Thursday
5. Monday falls on 4th April, 1988. What was the day on 3rd November, 1987?
   a) Monday b) Sunday c) Tuesday d) Wednesday
6. Today is 1st August. The day of the week is Monday. This is a leap year. The day of the week on this day after 3 years will be:
   a) Wednesday b) Thursday c) Friday d) Saturday
7. January 16, 1997 was a Thursday. What day of the week will it be on January 4, 2000?
   a) Tuesday b) Thursday c) Wednesday d) Friday
8. February 20, 1999 was Saturday. What day of the week was on December 30, 1997?
   a) Tuesday b) Monday
   c) Thursday d) Data inadequate
9. March 5, 1999 was on Friday, what day of the week will be on March 5, 2000?
   a) Monday b) Tuesday c) Sunday d) None of these

Answers

1. a; **Hint:** First find the day on 1st August, 1980.
1st August, 1980 means, '(1979 years + 7 months + 1 day)'.
Now 1600 years contain 0 odd day.
300 years contain 15 or 1 odd day.
{19 leap years + 60 ordinary years} = 38 + 60 or 98 or 0 odd day
Thus 1979 years contain 0 + 1 + 0 = 1 odd day.
Number of days from Jan., 1980 upto 1st Aug, 1980.
Jan Feb March April May June July Aug
31 + 29 + 31 + 30 + 31 + 30 + 31 + 1
= 214 days = 30 weeks + 4 days = 4 odd days.
Total number of odd days = 1 + 4 = 5.
So, on 1st Aug, 1980, it was 'Friday'.
So, 1st Monday in August, 1980 lies on 4th August.
∴ Monday falls on 4th, 11th, 18th, & 25th in August, 1980.

2. d; **Hint:** 1992 being a leap year, it has 2 odd days. So, the first day of the year
1993 will be two days beyond Wednesday. ie it will be Friday.

3. b; **Hint:** The year 1979 bieng an ordinary year, it has 1 odd day.
So, the day on 12th January 1980 is one day beyond the day on 12th January, 1979.
But, January 12, 1980 being Saturday
∴ January 12, 1979 was Friday.

4. c; **Hint:** The year 1984 being a leap year, it has 2 odd days.
So, the day on 2nd July, 1985 is two days beyond the day on 2nd July, 1984.
But, 2nd July 1985 was Wednesday.
∴ 2nd July, 1984 was Monday.

5. c; **Hint:** Counting the number of days after 3rd November, 1987 we have:
Nov Dec Jan Feb March April
days 27 + 31 + 31 + 29 + 31 + 4
= 153 days containing 6 odd days
ie, (7 – 6) = 1 day beyond the day on 4th April, 1988.
So, the day was Tuesday.

6. b; **Hint:** This being a leap year none of the next 3 years is a leap year. So, the day of the week will be 3 days beyond Monday ie, it will be Thursday.

7. a; **Hint:** First we look for the leap years during this period.
1997, 1998, 1999 are not leap years.
1998 and 1999 together have net 2 odd days.
No. of days remaining in 1997 = 365 – 16 = 349 days = 49 weeks 6 odd days.
January 4, 2000 gives 4 odd days.
∴ Total no. of odd days = 2 + 6 + 4 = 12 days = 7 days (1 week) + 5 odd days
Hence, January 4, 2000 will be 5 days beyond thursday ie it will be on Tuesday.

8. a; **Hint:** The year during this interval was 1998 and it was not a leap year. Now, we calculate the no. of odd days in 1999 up to February 19:

January 1999 gives	3 odd days
19 February 1999 gives	5 odd days
1998, being ordinary year, gives	1 odd day
In 1997, December 30 and 31 give	2 odd days

∴ total no. of odd days = 3 + 5 + 1 + 2 = 11 days = 4 odd days
Therefore, December 30, 1997 will fall 4 days before Saturday ie on Tuesday.

9. c; **Hint:** Year 2000 is a leap year.
No. of remaining days in 1999 =
365 – [31 days in January + 28 days in February + 5 days in March] = 301 days = 43 weeks ie 0 odd day.
No. of days passed in 2000 =
January (31 days) gives 3 odd days
February (29 days, being a leap year) gives 1 odd day
March (5 days) gives 5 odd days
∴ total no. of odd days = 0 + 3 + 1 + 5 = 9 days ie 2 odd days.
Therefore, March 5, 2000 will be two days beyond Friday, ie on Sunday.

Rule 2

To find the day of a week by the help of the number of odd days, when no reference day is given:

Working Rule

1. Count the net number of odd days on the given date.
2. In that case we count days according to number of odd days. See the table given below

Number of odd days	Days
0	Sunday
1	Monday
2	Tuesday
3	Wednesday
4	Thursday
5	Friday
6	Saturday

Note: **1.** First January, 1 AD was Monday. Therefore we must count days from Sunday ie Sunday for 0 odd day, Monday for 1 odd day, Tuesday for 2 odd days, and so on.

2. February in an ordinary year gives no odd day, but in a leap year gives one odd day.

Suppose someone asks you to find the day of the week on 12th January 1979.
12th Jan, 1979 means 1978 years + 12 days
Now, 1600 years have 0 odd day.
300 years have 5 × 3 = 15 ie 1 odd day
78 years have 59 ordinary years + 19 leap years
= 59 + 2 × 19 = 97 days = 13 weeks + 6 days
= **6 odd days**
Total number of odd days = 0 + 1 + 6 + 12 = 19
or **5 odd days**
So the day was **Friday** [See the table]

Illustrative Examples

Ex. 1: The first Republic Day of India was celebrated on 26th January 1950. What was the day of the week on that date?

Soln: Total no. of odd days = 1600 years have 0 odd day + 300 years have 1 odd day + 49 years (12 leap + 37 ordinary) have 5 odd days + 26 days of Jan have 5 odd days = 0 + 1 + 5 + 5 = 4 odd days
So, the day was Thursday.

Ex. 2: Mahatma Gandhi was born on 2 Oct 1869. The day of the week was.

Soln: 1600 years have 0 odd day
200 years have 2 × 5 = 10, ie, 3 odd days.
68 years contain 17 leap years and 51 ordinary years.
That is, 17 × 2 + 51 = 85 days, ie, 1 odd day.
In 1869, upto 2nd Oct, total number odd days = 31 (Jan) + 28 (Feb) + 31 (Mar) + 30 (Apr) + 31 (May) + 30 (Jun) + 31 (Jul) + 31 (Aug) + 30 (Sep) + 2 (Oct) = 275 days = 2 odd days
∴ total odd days = 0 + 3 + 1 + 2 = 6 odd days.
∴ the day was Saturday.

Other Method:
Whenever, the day of week for a year later than 1600 is asked, divide the years like (in this case)
2 Oct of 1869
= 1600 + 200 + 68 + Jan 1 to 2 Oct of 1869
= 1600 + 200 + 68 + 365 days – 2 Oct to 31 Dec of 1869
= 1600 + 200 + 68 + (365 – 90) days
No. of odd days = 0 + 3 + 1 (for 17 leap years & 51 ordinary years) + 2 = 6 odd days
∴ the day is Saturday.

Ex. 3: India got Independence on 15th August 1947. What was the day of the week?

Soln: 15 Aug 1947 = (1600 + 300 + 46) years + 1 Jan to 15 Aug of 1947

= (1600 + 300 + 46) years + 365 – 16
Aug to 31 Dec 1947
= (1600 + 300 + 46) years + (365 – 138) days
No. of odd days = 0 + 1 + 1 (from 11 leap years and 35 ordinary years) + 3 = 5 odd days.
∴ the day was **Friday**.

Remember the following table

Months	Odd days
Jan	3
Feb	0/1 (Ordinary/leap year)
Mar	3
Apr	2
May	3
Jun	2
Jul	3
Aug	3
Sep	2
Oct	3
Nov	2
Dec	3

Exercise

1. Find the day of the week on
(i) 16th July, 1776.
a) Monday b) Tuesday
c) Wednesday d) None of these
(ii) 12th January, 1979.
a) Saturday b) Friday
c) Wednesday d) Thursday
2. Today is Friday. After 62 days it will be:
a) Friday b) Thursday c) Saturday d) Monday
3. Smt Indira Gandhi died on 31st October, 1984. The day of the week was:
a) Monday b) Tuesday c) Wednesday d) Friday
4. The year next to 1988 having the same calendar as that of 1988 is:
a) 1990 b) 1992 c) 1993 d) 1995
5. The year next to 1991 having the same calendar as that of 1990 is:
a) 1998 b) 2001 c) 2002 d) 2003
6. What day of the week 20th June, 1837?
a) Monday b) Tuesday c) Thursday d) Friday
7. The year next to 1990 having the same calendar as that of 1990 is _____.
a) 1998 b) 2001 c) 2002 d) 2004

Answers

1. (i) b; **Hint:** 16th July, 1776 means
(1775 years + 6 months + 16 days)
Now, 1600 years have 0 odd days.
100 years have 5 odd days.
75 years contain 18 leap yeas & 57 ordinary years and therefore
(36 + 57) or 93 or 2 odd days.
∴ 1775 years give 0 + 5 + 2 = 7 and so 0 odd day.
Also number of days from 1st Jan., 1776 to 16th July, 1776
Jan Feb March April May June July
31 + 29 + 31 + 30 + 31 + 30 + 16
= 198 days = 28 weeks + 2 days = 2 odd days.
∴ Total number of odd days = 0 + 2 = 2.
Hence the day on 16th July, 1776 was 'Tuesday'.

(ii) b; **Hint:** 12th January, 1979 means, (1978 years + 12 days)
Now 1600 years have 0 odd day
300 years have 15 or 1 odd day
78 years have
{19 leap years + 59 ordinary years
= (38 + 59) or 97 odd days or 6 odd days
12 days of January has 5 odd days
Total number of odd days : 0 + 1 + 6 + 5
= 12 or 5 odd days.
So, the day was 'Friday'.

2. b; **Hint:** Each day of the week is repeated after 7 days.
∴ After 63 days, it would be Friday.
So, after 62 days, it would be Thursday.

3. c; **Hint:** 1600 years contain 0 odd day; 300 years contain 1 odd day.
Also, 83 years contain 20 leap years and 63 ordinary years and therefore (40 + 0) odd days ie, 5 odd days.
∴ 1983 years contain (0 + 1 + 5) ie, 6 odd days.
Number of days from Jan, 1984 to 31st Oct 1984.
= (31 + 29 + 31 + 30 + 31 + 30 + 31 + 31 + 30 + 31)
= 305 days = 4 odd days
∴ Total number of odd days = 6 + 4 = 10 ie 3 odd days.
So, 31st Oct, 1984 was Wednesday.

4. c; **Hint:** Starting with 1988, we go on counting the number of odd days till the sum is divisible by 7

Years	1988	1989	1990	1991	1992
Odd days	2	1	1	1	2

= 7 ie 0 odd day
∴ Calendar for 1993 is the same as that of 1988.

5. c; **Hint:** We go on counting the odd days from 1991 onwards till the sum is divisible by 7. The number of such days are 14 upto the year 2001.
So, the calendar for 1991 will be repeated in the year 2002.

6. b; **Hint:** 20th June, 1837 means "1836 complete years + first 5 months of the year 1837 + 20 days of June"
1600 years give no odd day
200 years give 3 odd days
36 years give 3 odd days.
[36 years contain 9 leap years and 27 ordinary years and therefore, (27 + 18 =) 45 odd days = 3 odd days].
∴ 1836 years give (0 + 3 + 3) = 6 odd days
Now, from first January to 20th June

we have,

	odd days
January	= 3
February	= 0
March	= 3
April	= 2
May	= 3
June	= 6
	17 ie 3 odd days.

∴ Total number of odd days = (6 + 3) = 9 odd days ie 2 odd days.

This means that the 20th June fell on the 2nd day commencing from Monday. Therefore the required day was **Tuesday**.

7. b; **Hint:** We go on counting the no. of odd days from 1990 onward till the sum is exactly divisible by 7.

The no. of such days are 14 upto the year 2000. So the calendar for 1990 will be repeated in the year 2001.

Note: No. of odd days = 1(1990) + 1(1991) + 2(1992) + 1(1993) + 1(1994) + 1(1995) + 2(1996) + 1(1997) + 1(1998) + 1(1999) + 0(2000) + 1(2001) + 1(2002) = 14 odd days.

Miscellaneous

1. Prove that the calendar for 1990 will serve for 2001 also.
2. Prove that the last day of a century can not be either Tuesday, Thursday or Saturday.
3. Prove that any date in March is the same day of the week as the corresponding date in November of that year.
4. How many times does the 29th day of the month occur in 400 consecutive years?
   a) 4400 times b) 4487 times c) 4496 times d) 4497 times

Answers

1. **Hint:** In order that the calendar for 1990 and 2001 be the same, 1st January of both the years must be on the same day of the week. For this, the total number of odd days between 31st Dec 1989 and 31st Dec 2000 must be zero.

Odd days are as under

Year	No. of odd days
1990	1
1991	1
1992	2
1993	1
1994	1
1995	1
1996	2
1997	1
1998	1
1999	1
2000 (leap year)	2

∴ total no. of odd days
= [1 + 1 + 2 + 1 + 1 + 1 + 2 + 1 + 1 + 1 + 2]
= 14 days ie 0 odd days.

Hence the result follows.

2. **Hint:** 1st Century, ie 100 years contain 76 ordinary years and 24 leap years and therefore, (76 + 48) or 124 odd days or 5 odd days.

∴ The last day of 1st century is 'Friday'.

Two Centuries, ie 200 years contain 152 ordinary years and 48 leap years and therefore (152 + 96) or 248 or 3 odd days.

∴ The last day of 2nd century is 'Wednesday'.

Three Centuries, ie 300 years contain 228 ordinary years and 72 leap years and therefore, (228 + 144) or 372 or 1 odd day.

∴ The last day of third century is 'Monday'.

Four Centuries, ie 400 years contain 303 ordinary years and 97 leap years and therefore, (303 + 194) or 497 or 0 odd day.

∴ The last day of 4th century is 'Sunday'.

Since the order is continually kept in successive cycles, we see that the last day of a century can not be Tuesday, Thursday or Saturday.

Note: The first day of a century must be either Monday, Tuesday, Thursday or Saturday.

3. **Hint:** In order to prove the required result, we have to show that the total number of odd days between last day of February and last day of October is zero.

Number of days between these dates are:

March April May June July Aug Sept Oct
31 + 30 + 31 + 30 + 31 + 31 + 30 + 31
= 245 days = 35 weeks = 0 odd day.

Hence, the result follows.

4. d; **Hint:** In 400 consecutive years there are 97 leap years. Hence in 400 consecutive years February has the 29th day 97 times, and the remaining 11 months have the 29th day 400 × 11 or 4400 times.

∴ the 29th day of the month occurs (4400 + 97)
= 4497 times.