7. TRIGONOMETRIC FUNCTIONS

Synopsis :

1. Let $\theta \in R$. Take an angle of measure θ radians in the standard position. Let P(x, y) be a point on the terminal side of the angle θ such that OP = r(> 0). Then

   i) $\frac{y}{r}$ is called sine of θ and it is denoted by $\sin\theta$.

   ii) $\frac{x}{r}$ is called cosine of θ and it is denoted by $\cos\theta$

   iii) $\frac{y}{x}$ $(x \neq 0)$ is called tangent of θ and it is denoted by $\tan\theta$.

   iv) $\frac{x}{y}$ $(y \neq 0)$ is called cotangent of θ and it is denoted by $\cot\theta$.

   v) $\frac{r}{x}$ $(x \neq 0)$ is called secant of θ and it is denoted by $\sec\theta$.

   vi) $\frac{r}{y}$ $(y \neq 0)$ is called cosecant of θ and it is denoted by $\operatorname{cosec}\theta$.

   These six functions (ratios) are called trigonometric functions (ratios).

2. $\sin\theta.\operatorname{cosec}\theta = 1$, $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$, $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

3. $\cos\theta.\sec\theta = 1$, $\cos\theta = \frac{1}{\sec\theta}$, $\sec\theta = \frac{1}{\cos\theta}$

4. $\tan\theta.\cot\theta = 1$, $\tan\theta = \frac{1}{\cot\theta}$, $\cot\theta = \frac{1}{\tan\theta}$

5. $\frac{\sin\theta}{\cos\theta} = \tan\theta$, $\frac{\cos\theta}{\sin\theta} = \cot\theta$

6. $\sin^2\theta + \cos^2\theta = 1$,

   $\sin^2\theta = 1 - \cos^2\theta$,

   $\cos^2\theta = 1 - \sin^2\theta$

7. $1+\tan^2\theta = \sec^2\theta$, $\tan^2\theta = \sec^2\theta - 1$, $\sec^2\theta - \tan^2\theta = 1$.

8. $1 + \cot^2\theta = \operatorname{cosec}^2\theta$, $\cot^2\theta = \operatorname{cosec}^2\theta - 1$,

   $\operatorname{cosec}^2\theta - \cot^2\theta = 1$.

9. $\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$.

10. $\text{cosec }\theta + \cot\theta = \dfrac{1}{\text{cosec}\theta - \cot\theta}$.

11. The values of the trigonometric functions of some standard angles :

θ	0	π/6	π/4	π/3	π/2	π	3π/2	2π
sin	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0	–1	0
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0	–1	0	1

12. Trigonometric functions of $2n\pi + \theta$; $n \in Z$

1) $\sin(2n\pi + \theta) = \sin\theta$, $\cos(2n\pi + \theta) = \cos\theta$

2) $\tan(2n\pi + \theta) = \tan\theta$, $\cot(2n\pi + \theta) = \cot\theta$

3) $\sec(2n\pi + \theta) = \sec\theta$, $\text{cosec}(2n\pi + \theta) = \text{cosec}\theta$

13. Trigonometric functions of $(-\theta)$, for all values of θ

1) $\sin(-\theta) = -\sin\theta$, 2) $\cos(-\theta) = \cos\theta$,

3) $\tan(-\theta) = -\tan\theta$, 4) $\cot(-\theta) = -\cot\theta$,

5) $\sec(-\theta) = \sec\theta$, 6) $\text{cosec}(-\theta) = -\text{cosec}\,\theta$

14. The values of trigonometric functions of any angle can be represented in terms of an angle in the first quadrant

Let $A = n.\frac{\pi}{2} \pm \theta$ where $n \in Z$, $0 \le \theta \le \frac{\pi}{2}$. Then

i) $\sin\left(n.\frac{\pi}{2} \pm \theta\right)$ $= \pm \sin\theta$, if n is even

$= \pm \cos\theta$, if n is odd

ii) $\cos\left(n.\frac{\pi}{2} \pm \theta\right)$ $= \pm \cos\theta$, if n is even

$= \pm \sin\theta$, if n is odd

iii) $\tan\left(n.\frac{\pi}{2} \pm \theta\right)$ $= \pm \tan\theta$, if n is even

$= \pm \cot\theta$, if n is odd

iv) $\cot\left(n.\frac{\pi}{2} \pm \theta\right)$ $= \pm \cot\theta$, if n is even

$= \pm \tan\theta$, if n is odd

v) $\sec\left(n.\frac{\pi}{2} \pm \theta\right)$ $= \pm \sec\theta$, if n is even

$= \pm \text{cosec}\,\theta$, if n is odd

vi) $\text{cosec}\left(n.\frac{\pi}{2} \pm \theta\right)$ $= \pm \text{cosec}\,\theta$, if n is even

$= \pm \sec\theta$, if n is odd